

Euclid's Elements

Definitions, Postulates, Common Notions, and Proposition 1.1

DEFINITIONS

Definition 1. A point is that which has no part.

Definition 2. A line is breadthless length.

Definition 3. The ends of a line are points.

Definition 4. A straight line is a line which lies evenly with the points on itself.

Definition 5. A surface is that which has length and breadth only.

Definition 6. The edges of a surface are lines.

Definition 7. A plane surface is a surface which lies evenly with the straight lines on itself.

Definition 8. A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.

Definition 9. And when the lines containing the angle are straight, the angle is called rectilinear.

Definition 10. When a straight line standing on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands.

Definition 11. An obtuse angle is an angle greater than a right angle.

Definition 12. An acute angle is an angle less than a right angle.

Definition 13. A boundary is that which is an extremity of anything.

Definition 14. A figure is that which is contained by any boundary or boundaries.

Definition 15. A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure equal one another.

Definition 16. And the point is called the center of the circle.

Definition 17. A diameter of the circle is any straight line drawn through the center and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.

Definition 18. A semicircle is the figure contained by the diameter and the circumference cut off by it. And the center of the semicircle is the same as that of the circle.

Definition 19. Rectilinear figures are those which are contained by straight lines, trilateral figures being those contained by three, quadrilateral those contained by four, and multilateral those contained by more than four straight lines.

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Definition 20. Of trilateral figures, an equilateral triangle is that which has its three sides equal, an isosceles triangle that which has two of its sides alone equal, and a scalene triangle that which has its three sides unequal.

Definition 21. Further, of trilateral figures, a right-angled triangle is that which has a right angle, an obtuse-angled triangle that which has an obtuse angle, and an acute-angled triangle that which has its three angles acute.

Definition 22. Of quadrilateral figures, a square is that which is both equilateral and right-angled; an oblong that which is right-angled but not equilateral; a rhombus that which is equilateral but not right-angled; and a rhomboid that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called trapezia.

Definition 23. Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

POSTULATES

Let the following be postulated:

Postulate 1. To draw a straight line from any point to any point.

Postulate 2. To produce a finite straight line continuously in a straight line.

Postulate 3. To describe a circle with any center and radius.

Postulate 4. That all right angles equal one another.

Postulate 5. That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

COMMON NOTIONS

Common notion 1. Things which equal the same thing also equal one another.

Common notion 2. If equals are added to equals, then the wholes are equal.

Common notion 3. If equals are subtracted from equals, then the remainders are equal.

Common notion 4. Things which coincide with one another equal one another.

Common notion 5. The whole is greater than the part.

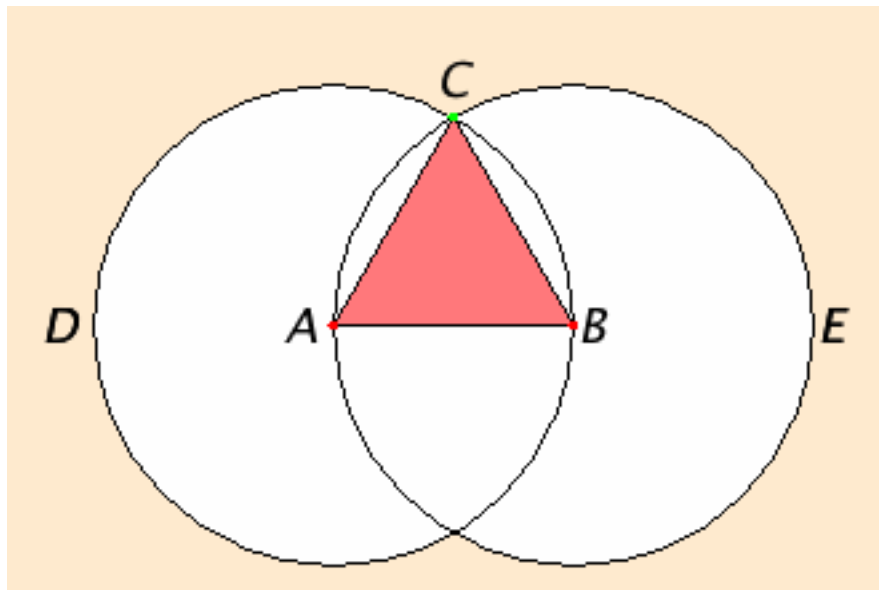
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PROPOSITION 1

To construct an equilateral triangle on a given finite straight line.

Let AB be the given finite straight line.



It is required to construct an equilateral triangle on the straight line AB .

Describe the circle BCD with center A and radius AB . Again describe the circle ACE with center B and radius BA . Join the straight lines CA and CB from the point C at which the circles cut one another to the points A and B .

Now, since the point A is the center of the circle CDB , therefore AC equals AB . Again, since the point B is the center of the circle CAE , therefore BC equals BA .

But AC was proved equal to AB , therefore each of the straight lines AC and BC equals AB .

And things which equal the same thing also equal one another, therefore AC also equals BC .

Therefore, the three straight lines AC , AB , and BC equal one another.

Therefore, the triangle ABC is equilateral, and it has been constructed on the given finite straight line AB .

Q.E.F.